CARMA processes: properties, applications and inference

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Abstract

Just as ARMA processes, $(Y_n)_{n \in \mathbb{Z}}$, defined by difference equations, play a central role in the representation of stationary time series with discrete time parameter, CARMA (continuous-time ARMA) processes, $(Y(t))_{t \in \mathbb{R}}$, defined in terms of stochastic differential equations, play an analogous role in the representation of stationary time series with continuous time parameter. Lévy-driven CARMA processes permit the modelling of heavy-tailed and asymmetric time series and incorporate both distributional and sample-path information. In particular the sample-paths may exhibit jumps (see [1]). Continuous-time models are of interest both in their own right and in the modelling of discrete time series data which are not regularly spaced in time. Irregular spacing of data is especially relevant in the modelling of stationary random fields ([3]).

Second-order CARMA processes have been of interest to physicists and engineers for many years and have been analyzed, primarily from a spectral point of view, by researchers as early as Doob [4]. In this talk we focus on CARMA processes which may not have finite second-order moments and to which spectral analysis may therefore not apply. We shall define these processes and examine their basic properties.

In the last fifteen years there has been a resurgence of interest in these processes and in continuous-time processes more generally, partly as a result of the very successful application of stochastic differential equation models to problems in finance. The proliferation of high-frequency data, especially in fields such as finance and turbulence, has stimulated interest also in the connections between CARMA processes and the discrete-time processes obtained by sampling them at regular intervals. We shall make these connections explicit ([2]) and show how the results can be applied to inference for CARMA processes based on regularly sampled data, showing in particular the effect of sampling frequency on the behavior of the parameter estimators.

References

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