A Market Model with error terms following symmetric-asymmetric distribution family and Decomposition of Jensen's Alpha : Theoretical and empirical study with Stock market data

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(This is a joint study with Navruzbek Karamatov, Tohoku University.)

Motivation

Our main interest is firstly in the shape of the residual distribution which is seen in stock market data analysis based on a simple linear regression mode, and secondly the comparison of two estimation method for the slope parameter β: Usual OLS and Estimates based on Rank statistics (R-estimates in short).

As it is known well, R-estimate can estimate β without necessarily estimating the intercept parameter α. Also it is Robust against outliers.

In the performance study for stock portfolios, Jensen's α is of main interest. Our study reveals how this is related to the degree of asymmetry in the distribution of residuals.

Model and Estimation Method.

Error terms ε_i are i.i.d and η_i have a distribution $G(x-\mu)$ in the following simple linear regression model which is called Market Model in the field of Finance.

$Y_i = \alpha + \beta X_i + \varepsilon_i$ …… (1) and $\eta_i = Y_i - \beta X_i = \alpha + \varepsilon_i$ …..(2). for $i = 1, \ldots, n$.

GLAM (Generalized Lehmann's Alternative Model) is a semi-parametric model and its parameter can be estimated based on rank statistics (Miura and Tsukahara (1993)).

Let Θ be an interval in real line.

A function $h(t;\theta)$ for $t \in (0,1)$ and $\theta \in \Theta$ which satisfies the following (1) and (2) is called the Generalized Lehmann's Alternative model:

(1) $h = (0;\theta) = 0$ and $h(1;\theta) = 1$ for any $\theta \in \Theta$. $h(t;\theta)$ is strictly monotone function of t.

(2) There exists $\theta* \in \Theta$ such that $h(t;\theta*) = t$ for $t \in (0,1)$.

And for $\theta_1<\theta_2$, $h(t;\theta_1) < h(t;\theta_2)$ for all $t \in (0,1)$.

Then, we assume that η_i follows $G(x : \mu,\theta) = h(F(x-\mu);\theta) = 1-(1-F(x-\mu))^{\theta}$. F is unknown except its symmetry. The parameters θ and μ are estimated using the residuals (after estimating β).

Results.

We used the stock prices data in three markets: Tokyo, New York and London. The shape of residuals distribution brought by the two methods look different or about the same depending on the individual stock and the time period. A decomposition of α into a sum of μ and "asymmetry effect" will be shown among many other results.