TEST THEORY FOR NOISILY OBSERVED DIFFUSION PROCESSES

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We study the d-dimensional diffusion process $\{X_t\}_{t\geq 0}$ defined by the following stochastic differential equation:

$$\mathrm{d}X_t = b\left(X_t,\beta\right)\mathrm{d}t + a\left(X_t,\alpha\right)\mathrm{d}w_t, X_0 = x_0,$$

where $\{w_t\}_{t\geq 0}$ is an r-dimensional Wiener process, x_0 is a d-dimensional random variable independent of $\{w_t\}$, $\alpha \in \Theta_1 \subset \mathbf{R}^{m_1}$ and $\beta \in \Theta_2 \subset \mathbf{R}^{m_2}$ are unknown parameters, $\Theta := \Theta_1 \times \Theta_2$ is the compact and convex parameter space, $a : \mathbf{R}^d \times \Theta_1 \to \mathbf{R}^d \otimes \mathbf{R}^r$ and $b : \mathbf{R}^d \times \Theta_2 \to \mathbf{R}^d$ are known functions.

Our interest is to test hypotheses about α and β based on noisy and discrete observation of $\{X_t\}$. The sequence of observation $\{Y_{ih_n}\}_{i=0,\ldots,n}$ is defined as

$$Y_{ih_n} := X_{ih_n} + \Lambda^{1/2}\varepsilon_{ih_n}, i = 0,\ldots,n,$$

where h_n is the discretisation step satisfying $h_n \to 0$ and $T_n := nh_n \to \infty$ as $n \to \infty$, $\{\varepsilon_{ih_n}\}_{i=0,\ldots,n}$ is the i.i.d. sequence of $\mathbf{R}^d$-valued random variables with the properties (i) independence among components, (ii) marginal densities symmetric w.r.t. 0, (iii) $\mathbf{E}\left[\varepsilon_0\right] = \mathbf{0}$, and (iv) $\mathrm{Var}\left(\varepsilon_0\right) = I_d$, and $\Lambda \in \mathbf{R}^d \otimes \mathbf{R}^d$ is a positive semi-definite matrix defining the variance of noise term.

Nakakita and Uchida (2019) propose Gaussian-type adaptive quasi-likelihood functions $\mathbb{H}_{1,n}\left(\alpha|\left\{Y_{ih_n}\right\}\right)$ and $\mathbb{H}_{2,n}\left(\beta|\left\{Y_{ih_n}\right\}\right)$ for α and β respectively, and show consistency and asymptotic normality of the maximum-likelihood-type estimators $\hat{\alpha}_n$ and $\hat{\beta}_n$ which are random variables satisfying $\sup_{\alpha\in\Theta_1}\mathbb{H}_{1,n}\left(\alpha|\left\{Y_{ih_n}\right\}\right) = \mathbb{H}_{1,n}\left(\hat{\alpha}_n|\left\{Y_{ih_n}\right\}\right)$ and $\sup_{\beta\in\Theta_2}\mathbb{H}_{2,n}\left(\beta|\left\{Y_{ih_n}\right\}\right) = \mathbb{H}_{2,n}(\hat{\beta}_n|\left\{Y_{ih_n}\right\})$.

In this study, we utilise the asymptotic normality of those estimators to compose test statistics for the following form of statistical hypotheses:

$$H_0 : \alpha^{(1)} = \cdots = \alpha^{(\lambda_1)} = 0,\ H_1 : \text{not } H_0,$$

where $\lambda_1 \in \{1,\ldots,m_1\}$, and

$$H_0 : \beta^{(1)} = \cdots = \beta^{(\lambda_2)} = 0,\ H_1 : \text{not } H,$$

where $\lambda_2 \in \{1,\ldots,m_2\}$. The test statistics we study are classified into three types: likelihood-ratio-type; Wald-type; and Rao-type. We see asymptotic behaviours of the test statistics and examine the actual performance of them in computational simulation. In addition, the real data analysis for wind data with high-frequency observation is shown as an application of the study.

References

Nakakita, S. H. and Uchida, M. (2019). Inference for ergodic diffusions plus noise. *Scandinavian Journal of Statistics*, 46(2):470–516.