PROBABILITY WEIGHTING FUNCTION DERIVED FROM HYPERBOLIC TIME DISCOUNTING: PSYCHOMETRIC MODELS AND THEIR EMPIRICAL TESTING

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A probability weighting function ($w(p)$) is considered to be a nonlinear function of objective probability (p) in behavioral decision theory. There are several psychometric models for representing probability weighting functions (e.g., Tversky and Kahneman, 1992; Prelec, 1998; Takahashi, 2011). However, most of the proposed models are not related to traditional psychological theories such as psychophysics and learning theory, with the exception of a study by Takahashi (2011). Takahashi (2011) combined psychophysical theory with Cajueiro's (2006) q-exponential function for explaining time discounting, proposed a new general model, and then derived Prelec's (1998) probability weighting function as a special case. The merit of combining a probability weighting model with a time discounting model is to create an integrated human decision model. The time discounting model and decision under risk, whose probability distribution is known, are both important areas in behavioral decision research. However, there seems to be a strong connection between them.

This study proposes a psychometric model of probability weighting functions derived from a hyperbolic time discounting model and a geometric distribution. The aim of the study is to show probability weighting functions from the point of view of waiting time for a decision maker. Since the expected value of a geometric distributed random variable X is $1/p$, we formulized the probability weighting function of the expected value model for hyperbolic time discounting as $w(p) = (1 - k \log p)^{-1}$. Moreover, the probability weighting function is derived from Lowenstein and Prelec's (1992) generalized hyperbolic time discounting model. The latter model is proved to be equivalent to the hyperbolic-logarithmic weighting function considered by Prelec (1998) and Luce (2001). In this study, we derive a model from the generalized hyperbolic time discounting model assuming Fechner's psychophysical law of time and a geometric distribution of trials, and then present an axiomatic system for the generalized hyperbolic model. In addition, we develop median models of hyperbolic time discounting and generalized hyperbolic time discounting. To illustrate the fitness of each model, a psychological experiment was conducted for assessing the probability weighting and value functions at the level of the individual participants. The results indicated that the expected value model of generalized hyperbolic discounting fit better than previous psychometric models.

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