Improved Empirical Likelihood Inference for Average Treatment Effects

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Abstract

The purpose of this paper is to show how improved semiparametric inference can be constructed by empirical likelihood (EL). Usual Wald-type inference needs undersmoothing for estimating the nonparametric component to ensure $\sqrt{n}$-consistency of the estimator of the parameters. On the other hand, plug-in empirical likelihood inference is not asymptotically pivotal even with undersmoothing. We propose an empirical likelihood statistic based on the influence function for average treatment effects. We show that it is asymptotically pivotal, having chi-square limiting distributions without undersmoothing. A simulation study is undertaken to compare the proposed EL, Wald-type and plug-in EL inference.

Let $Y_i(1)$ and $Y_i(0)$ denote potential outcomes of unit i with and without exposure to a treatment. Let $T_i \in \{0,1\}$ be an indicator variable for the treatment such that $T_i = 1$ if unit i is exposed to the treatment and $T_i = 0$ otherwise. We observe $Z_i = (Y_i, X_i', T_i)'$ where

$$Y_i = T_i Y_i(1) + (1 - T_i) Y_i(0),$$

is the realized outcome, and X_i is a r-dimensional vector of covariates. We are interested in the average treatment effect (ATE) parameter

$$\theta = E\left[Y_i(1) - Y_i(0)\right]. \tag{1}$$

In this paper, we propose to construct empirical likelihood ratios based on influence functions.

$$\ell_{IF}(\theta) = -2 \sup_{\{w_i\}_{i=1}^n} \left\{ \sum_{i=1}^n \log(nw_i) : w_i \geq 0,\ \sum_{i=1}^n w_i = 1, \sum_{i=1}^n w_i \tilde{g}(Z_i, \theta, \hat{p}(X_i)) = 0 \right\}$$

where

$$\tilde{g}(Z_i, \theta, \hat{p}(X_i)) = \left(\frac{Y_i T_i}{\hat{p}(X_i)} - \frac{Y_i(1 - T_i)}{1 - \hat{p}(X_i)} - \theta \right) + \hat{g}_p(Z_i, \theta, \hat{p}(X_i))(T_i - \hat{p}(X_i)),$$

and $\hat{g}_p(Z_i, \theta, \hat{p}(X_i)) = \sum_{j=1}^n w_{ji} \left[-\frac{Y_j T_j}{\hat{p}(X_j)^2} - \frac{Y_j(1-T_j)}{(1-\hat{p}(X_j))^2} \right]$.

Theorem 1. *Under Assumptions,*

$$\ell_{IF}(\theta_0) \xrightarrow{d} \chi^2(1).$$